Exam Seat No:________________ Enrollment No:___________________

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 5SC02MTE3 **Summer Examination-2014** **Date: 20 /06/2014**

Subject Name:- Number Theory

Branch/Semester:-M.Sc(Mathematics)/II **Time:02:00 To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places
(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) For integers a, b, c, if $c|a$ & $c|b$, then prove that $c|(ma + nb)$, $\forall\, m, n \in \mathbb{Z}$. (02)

b) If k is any positive integer, then prove that $k^2 + k + 1$ is not a square number. (02)

c) Define: Perfect number with example. (01)

d) If p is prime number and $p|ab$, either $p|a$ or $p|b$. Determine whether the statement is true or false. (01)

e) Congruence is an equivalent relation. Determine whether the statement is true or false. (01)

Q-2 a) In usual notations prove that, $[a, b](a, b) = ab$. (05)

b) Prove that every positive integer greater than one can be expressed uniquely as a product of prime, up to the order of the factor. (05)

c) If $p_n$is the n^{th}prime numbers, prove that $p_n < 2^{2^n}, \forall\, n$. (04)

OR

Q-2 a) Prove that $S(a) < a\sqrt{a}, \forall\, a > 2$. (05)

b) If m is composite integer and $n_m = 111 \ldots 111$ m times, prove that n_m is also composite number. (05)

c) If $k > 0$ is a common multiple a and b, prove that $\left(\frac{k}{a}, \frac{k}{b}\right) = \frac{k}{[a,b]}$. (04)

Q-3 a) Prove that the integer p is prime if and only if (05)

$$(p - 1)! + 1 \equiv 0\ (mod\ p).$$

b) State Chinese remainder theorem. Solve the system of three congruences (05)

$$x \equiv 2(mod\ 3), x \equiv 3(mod\ 5), x \equiv 2(mod\ 7)$$

c) State and prove Euler's theorem. (04)

OR

Q-3 a) In usual notation, prove that $p(n!) = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \cdots + \left[\frac{n}{p^m}\right]$, where $p^m \le n < p^{m+1}$ (05)

b) Prove that the necessary and sufficient condition for a positive integer n can be divided by 3 is that the sum of it digit is divisible by three. Is 57349896 divisible by 3? Justify. (05)

c) Define: Mobious function. Show that Mobious function is multiplicative. (04)

SECTION-II

Q-4 a) Express the rational number $\frac{19}{51}$ in finite simple continue fraction. (02)

b) If p, q is a positive solution of $x^2 - dy^2 = 1$, prove that $\frac{p}{q}$ is a convergent of the continued fraction expansion of $\sqrt{d}$. (02)

c) The value of any infinite continue fraction is an irrational number. Determine whether the statement is true or false. (01)

d) State Fermat's Last Theorem. (01)

e) Define: Algebraic number. (01)

Q-5 a) Prove that any rational number can be written as a finite simple continued fraction. (05)

b) Prove that general integer solution of $x^2 + y^2 = z^2$ with $x, y, z > 0$; $(x, y) = 1, y$ is even, is given by $x = a^2 - b^2, y = 2ab,\ z = a^2 + b^2$, where $a > b > 0, (a, b) = 1$ and one of a, b is odd and the other is even. (05)

c) Find the positive integer solution of the equation $7x + 19y = 213$. (04)

OR

Q-5 a) Prove that the k^{th} convergent of a simple continued fraction $[a_0; a_1, a_2, \dots, a_n]$ has the value $c_k = \frac{p_k}{q_k}, 0 \le k \le n$. (05)

b) Find the positive integer solution of the equation $19\,x + 20y = 1909$. (05)

c) Prove that for any positive integer n, the Diophantine equation $x^n + y^n = z^n$ has no positive integer solution less than n. (04)

Q-6 a) If $C_k = \frac{p_k}{q_k}$ is the k^{th} convergent of finite simple continued fraction $[a_0; a_1, a_2, \dots, a_n]$, then prove that (05)

$$p_k q_{k-1} - q_k p_{k-1} = (-1)^{k-1}, 1 \le k \le n\,.$$

b) If p is prime and $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$, a_n is incongruent to 0 modulo p, is a polynomial of degree $n \ge 1$ with integral coefficients, prove that $f(x) \equiv 0 (mod\, p)$ has at most n incongruent solutions modulo p. (05)

c) Find the unique irrational number represented by the infinite continued fraction $[3; 6, \overline{1, 4}]$. (04)

OR

Q-6 a) Prove that the product of two primitive polynomial is primitive. (05)

b) If $\frac{p_k}{q_k}$ are the convergents of the continuous fraction expansion of $\sqrt{d}$, prove that $p_k^2 - dq_k^2 = (-1)^{k+1}\, t_{k+1}$ where $t_{k+1} > 0, k = 0,1,2, \dots$ (05)

c) Solve the linear Diophantine equation $172\,x + 20y = 1000$ by using simple continued fractions. (04)

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